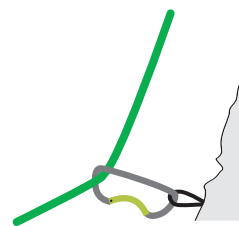


## UNITS, MEASUREMENTS AND CONSTANTS



Measurements are comparisons. The standard used for the comparison is called a *unit*. Many different systems of units have been used throughout the world. Unit systems are standards; they always confer a lot of power to the organisation in charge of them, as can be seen most clearly in the computer industry; in the past the same applied to measurement units. To avoid misuse by authoritarian institutions, to eliminate at the same time all problems with differing, changing and irreproducible standards, and – this is not a joke – to simplify tax collection, already in the 18th century scientists, politicians and economists have agreed on a set of units. It is called the *Système International d’Unités*, abbreviated *SI*, and is defined by an international treaty, the ‘Convention du Mètre’. The units are maintained by an international organisation, the ‘Conférence Générale des Poids et Mesures’, and its daughter organisations, the ‘Commission Internationale des Poids et Mesures’ and the ‘Bureau International des Poids et Mesures’, which all originated in the times just before the French revolution.

Ref. 975

All SI units are built from seven *base units* whose official definitions, translated from French into English, are the following, together with the date of their formulation:

- ‘The *second* is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom.’ (1967)\*
- ‘The *metre* is the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second.’ (1983)
- ‘The *kilogram* is the unit of mass; it is equal to the mass of the international prototype of the kilogram.’ (1901)\*
- ‘The *ampere* is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to  $2 \cdot 10^{-7}$  newton per metre of length.’ (1948)
- ‘The *kelvin*, unit of thermodynamic temperature, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.’ (1967)\*
- ‘The *mole* is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12.’ (1971)\*
- ‘The *candela* is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency  $540 \cdot 10^{12}$  hertz and has a radiant intensity in that direction

of (1/683) watt per steradian.’ (1979)\*

Note that both time and length units are defined as certain properties of a standard example of motion, namely light. This is an additional example making the point that the observation of motion as the fundamental type of change is a *prerequisite* for the definition and construction of time and space. By the way, the proposal of using light was made already in 1827 by Jacques Babinet.\*

From these basic units, all other units are defined by multiplication and division. In this way, all SI units have the following properties:

- They form a system with *state-of-the-art precision*; all units are defined in such a way that the precision of their definition is higher than the precision of commonly used measurements. Moreover, the precision of the definitions are regularly improved. The present relative uncertainty of the definition of the second is around  $10^{-14}$ , for the metre about  $10^{-10}$ , for the ampere  $10^{-7}$ , for the kilogram about  $10^{-9}$ , for the kelvin  $10^{-6}$ , for the mole less than  $10^{-6}$  and for the candela  $10^{-3}$ .

- They form an *absolute* system; all units are defined in such a way that they can be reproduced in every suitably equipped laboratory, independently, and with high precision. This avoids as much as possible any misuse by the standard setting organisation. (At present, the kilogram, still defined with help of an artefact, is the last exception to this requirement; extensive research is under way to eliminate this artefact from the definition – an international race that will take a few more years. A definition can be based only on two ways: counting particles or fixing  $\hbar$ . The former can be achieved in crystals, the latter using any formula where  $\hbar$  appears, such as the de Broglie wavelength, Josephson junctions, etc.)

- They form a *practical* system: base units are adapted to daily life quantities. Frequently used units have standard names and abbreviations. The complete list includes the seven base units, the derived, the supplementary and the admitted units:

The *derived* units with special names, in their official English spelling, i.e. without capital letters and accents, are:

name	abbreviation & definition	name	abbreviation & definition
hertz	Hz = 1/s	newton	N = kg m/s <sup>2</sup>
pascal	Pa = N/m <sup>2</sup> = kg/m s <sup>2</sup>	joule	J = Nm = kg m <sup>2</sup> /s <sup>2</sup>
watt	W = kg m <sup>2</sup> /s <sup>3</sup>	coulomb	C = As
volt	V = kg m <sup>2</sup> /As <sup>3</sup>	farad	F = As/V = A <sup>2</sup> s <sup>4</sup> /kg m <sup>2</sup>
ohm	$\Omega$ = V/A = kg m <sup>2</sup> /A <sup>2</sup> s <sup>3</sup>	siemens	S = 1/ $\Omega$
weber	Wb = Vs = kg m <sup>2</sup> /As <sup>2</sup>	tesla	T = Wb/m <sup>2</sup> = kg/As <sup>2</sup> = kg/Cs
henry	H = Vs/A = kg m <sup>2</sup> /A <sup>2</sup> s <sup>2</sup>	degree Celsius *	°C

\* The international prototype of the kilogram is a platinum–iridium cylinder kept at the BIPM in Sèvres, in France. For more details on the levels of the caesium atom, consult a book on atomic physics. The Celsius scale of temperature  $\theta$  is defined as:  $\theta/^\circ\text{C} = T/\text{K} - 273.15$ ; note the small difference with the number appearing in the definition of the kelvin. When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles. In its definition, it is understood that the carbon 12 atoms are unbound, at rest and in their ground state. The frequency of the light in the definition of the candela corresponds to 555.5 nm, i.e. green colour, and is the wavelength for which the eye is most sensitive.

\* Jacques Babinet (1794–1874), French physicist who published important work in optics.

name	abbreviation & definition	name	abbreviation & definition
lumen	lm = cd sr	lux	lx = lm/m <sup>2</sup> = cd sr/m <sup>2</sup>
becquerel	Bq = 1/s	gray	Gy = J/kg = m <sup>2</sup> /s <sup>2</sup>
sievert	Sv = J/kg = m <sup>2</sup> /s <sup>2</sup>	katal	kat = mol/s

We note that in all definitions of units, the kilogram only appears to the powers of 1, 0 and -1. The final explanation for this fact appeared only recently.

P. 896

The *radian* (rad) and the *steradian* (sr) are *supplementary SI units* for angle, defined as the ratio of arc length and radius, and for solid angle, defined as the ratio of the subtended area and the square of the radius, respectively.

The *admitted non-SI units* are *minute, hour, day* (for time), *degree*  $1^\circ = \pi/180$  rad, *minute*  $1' = \pi/10\,800$  rad, *second*  $1'' = \pi/648\,000$  rad (for angles), *litre* and *tonne*.

All other units are to be avoided.

All SI units are made more practical by the introduction of standard names and abbreviations for the powers of ten, the so-called *prefixes*:\*

name	abbr.	name	abbr.	name	abbr.	name	abbr.
10 <sup>1</sup>	deca da	10 <sup>-1</sup>	deci d	10 <sup>18</sup>	Exa E	10 <sup>-18</sup>	atto a
10 <sup>2</sup>	hecto h	10 <sup>-2</sup>	centi c	10 <sup>21</sup>	Zetta Z	10 <sup>-21</sup>	zepto z
10 <sup>3</sup>	kilo k	10 <sup>-3</sup>	milli m	10 <sup>24</sup>	Yotta Y	10 <sup>-24</sup>	yocto y
10 <sup>6</sup>	Mega M	10 <sup>-6</sup>	micro $\mu$	unofficial:		Ref. 977	
10 <sup>9</sup>	Giga G	10 <sup>-9</sup>	nano n	10 <sup>27</sup>	Xenta X	10 <sup>-27</sup>	xenno x
10 <sup>12</sup>	Tera T	10 <sup>-12</sup>	pico p	10 <sup>30</sup>	Wekta W	10 <sup>-30</sup>	weko w
10 <sup>15</sup>	Peta P	10 <sup>-15</sup>	femto f	10 <sup>33</sup>	Vendekta V	10 <sup>-33</sup>	vendeko v
				10 <sup>36</sup>	Udekta U	10 <sup>-36</sup>	udeko u

- SI units form a *complete* system; they cover in a systematic way the complete set of observables of physics. Moreover, they fix the units of measurements for physics and for all other sciences as well.

- They form a *universal* system; they can be used in trade, in industry, in commerce, at home, in education and in research. They could even be used by other civilisations, if they existed.

- They form a *coherent* system; the product or quotient of two SI units is also a SI unit. This means that in principle, the same abbreviation ‘SI’ could be used for every SI unit.

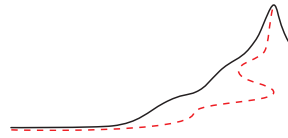
\* Some of these names are invented (yocto to sound similar to Latin octo ‘eight’, zepto to sound similar to Latin septem, yotta and zetta to resemble them, exa and peta to sound like the Greek words of six and five, the unofficial ones to sound similar to the Greek words for nine, ten, eleven and twelve), some are from Danish/Norwegian (atto from atten ‘eighteen’, femto from femten ‘fifteen’), some are from Latin (from mille ‘thousand’, from centum ‘hundred’, from decem ‘ten’, from nanus ‘dwarf’), some are from Italian (from piccolo ‘small’), some are Greek (micro is from μικρός ‘small’, deca/deka from δέκα ‘ten’, hecto from ἑκατόν ‘hundred’, kilo from χίλιοι ‘thousand’, mega from μέγας ‘large’, giga from γίγας ‘giant’, tera from τέρας ‘monster’).

Translate: I was caught in such a traffic jam that I needed a microcentury for a picoparsec and that my car’s fuel consumption was two tenths of a square millimetre.

Challenge 1299 e

Christoph Schiller

# MOTION MOUNTAIN



A hike beyond space and time  
along the concepts of modern physics

## To the kind reader

In exchange for getting this section for free, I ask you to send a short email that comments on one or more of the following:

- Which figures could be added?
- What was hard to understand?
- What could be improved or left out?

Most welcome of all is support on the specific points listed on the [www.motionmountain.net/support.html](http://www.motionmountain.net/support.html) web page. Thank you in advance for your input, also in the name of all other readers.

Like the whole of this physics text, also this section lives and grows through the feedback from readers like you, who help to improve and to complete it. For a particularly useful contribution you will be mentioned in the foreword, or receive a reward, or both. But above all, enjoy the reading.

C. Schiller, [mm@motionmountain.net](mailto:mm@motionmountain.net)

The SI units are not the only possible set that fulfils all these requirements, but they form the only existing system doing so.\*

We remind that since every measurement is a comparison with a standard, any measurement requires matter to realise the standard (yes, even for the speed standard) and radiation to achieve the comparison. Our concept of measurement thus assumes that matter and radiation exist and can be clearly separated from each other.

P. 850

### Planck's natural units

Since the exact form of many equations depends on the used system of units, theoretical physicists often use unit systems optimised for producing simple equations. In microscopic physics, the system of *Planck's natural units* is frequently used. They are automatically introduced by setting  $c = 1$ ,  $\hbar = 1$ ,  $G = 1$ ,  $k = 1$ ,  $\epsilon_0 = 1/4\pi$  and  $\mu_0 = 4\pi$  in equations written in SI units. Planck units are thus defined from combinations of fundamental constants; those corresponding to the fundamental SI units are given in the table.\*\* The table is also useful for converting equations written in natural units back to SI units; every quantity  $X$  is substituted by  $X/X_{\text{Pl}}$ .

**Table 69** Planck's natural units

Name	definition	value
<b>Basic units</b>		
the Planck length	$l_{\text{Pl}} = \sqrt{\hbar G/c^3}$	$= 1.616\,0(12) \cdot 10^{-35} \text{ m}$
the Planck time	$t_{\text{Pl}} = \sqrt{\hbar G/c^5}$	$= 5.390\,6(40) \cdot 10^{-44} \text{ s}$
the Planck mass	$m_{\text{Pl}} = \sqrt{\hbar c/G}$	$= 21.767(16) \mu\text{g}$
the Planck current	$I_{\text{Pl}} = \sqrt{4\pi\epsilon_0 c^6/G}$	$= 3.479\,3(22) \cdot 10^{25} \text{ A}$
the Planck temperature	$T_{\text{Pl}} = \sqrt{\hbar c^5/Gk^2}$	$= 1.417\,1(91) \cdot 10^{32} \text{ K}$
<b>Trivial units</b>		
the Planck velocity	$v_{\text{Pl}} = c$	$= 0.3 \text{ Gm/s}$
the Planck angular momentum	$L_{\text{Pl}} = \hbar$	$= 1.1 \cdot 10^{-34} \text{ Js}$
the Planck action	$S_{\text{aPl}} = \hbar$	$= 1.1 \cdot 10^{-34} \text{ Js}$

\* Most non-SI units still in use in the world are of Roman origin: the mile comes from 'milia passum' (used to be one thousand strides of about 1480 mm each; today a nautical mile, after having been defined as minute of arc, is exactly 1852 m), inch comes from 'uncia/onzia' (a twelfth – now of a foot); pound (from pondere 'to weigh') is used as a translation of 'libra' – balance – which is the origin of its abbreviation *lb*; even the habit of counting in dozens instead of tens is Roman in origin. These and all other similarly funny units – like the system in which all units start with 'f' and which uses furlong/fortnight as unit for velocity – are now officially defined as multiples of SI units.

\*\* The natural units  $x_{\text{Pl}}$  given here are those commonly used today, i.e. those defined using the constant  $\hbar$ , and not, as Planck originally did, by using the constant  $h = 2\pi\hbar$ . A similar, additional freedom of choice arises for the electromagnetic units, which can be defined with other factors than  $4\pi\epsilon_0$  in the expressions; for example, using  $4\pi\epsilon_0\alpha$ , with the *fine structure constant*  $\alpha$ , gives  $q_{\text{Pl}} = e$ . For the explanation of the numbers between brackets, the standard deviations, see page 939.

Name	definition	value
the Planck entropy	$S_{ePl} = k$	= 13.8 yJ/K
<b>Composed units</b>		
the Planck mass density	$\rho_{Pl} = c^5/G^2\hbar$	= $5.2 \cdot 10^{96}$ kg/m <sup>3</sup>
the Planck energy	$E_{Pl} = \sqrt{\hbar c^5/G}$	= 2.0 GJ = $1.2 \cdot 10^{28}$ eV
the Planck momentum	$p_{Pl} = \sqrt{\hbar c^3/G}$	= 6.5 Ns
the Planck force	$F_{Pl} = c^4/G$	= $1.2 \cdot 10^{44}$ N
the Planck power	$P_{Pl} = c^5/G$	= $3.6 \cdot 10^{52}$ W
the Planck acceleration	$a_{Pl} = \sqrt{c^7/\hbar G}$	= $5.6 \cdot 10^{51}$ m/s <sup>2</sup>
the Planck frequency	$f_{Pl} = \sqrt{c^5/\hbar G}$	= $1.9 \cdot 10^{43}$ Hz
the Planck electric charge	$q_{Pl} = \sqrt{4\pi\epsilon_0 c\hbar}$	= 1.9 aC = 11.7 e
the Planck voltage	$U_{Pl} = \sqrt{c^4/4\pi\epsilon_0 G}$	= $1.0 \cdot 10^{27}$ V
the Planck resistance	$R_{Pl} = 1/4\pi\epsilon_0 c$	= 30.0 $\Omega$
the Planck capacitance	$C_{Pl} = 4\pi\epsilon_0 \sqrt{\hbar G/c^3}$	= $1.8 \cdot 10^{-45}$ F
the Planck inductance	$L_{Pl} = (1/4\pi\epsilon_0) \sqrt{\hbar G/c^7}$	= $1.6 \cdot 10^{-42}$ H
the Planck electric field	$E_{Pl} = \sqrt{c^7/4\pi\epsilon_0 \hbar G^2}$	= $6.5 \cdot 10^{61}$ V/m
the Planck magnetic flux density	$B_{Pl} = \sqrt{c^5/4\pi\epsilon_0 \hbar G^2}$	= $2.2 \cdot 10^{53}$ T

The natural units are important for another reason: whenever a quantity is sloppily called ‘infinitely small (or large)’, the correct expression is ‘small (or large) as the corresponding Planck unit’. As explained in special relativity, general relativity and quantum theory, the third part, this substitution is correct because almost all Planck units provide, within a factor of the order 1, the extreme value for the corresponding observable. Unfortunately, these factors have not entered the mainstream yet; if  $G$  is substituted by  $4G$ ,  $\hbar$  by  $\hbar/2$  and  $4\pi\epsilon_0$  by  $8\pi\epsilon_0\alpha$  in all formulae, the exact extremal value for each observable in nature are obtained. These extremal values are the *true natural units*. Exceeding extremal values is possible only for extensive quantities, i.e. for those quantities for which many particle systems can exceed single particle limits, such as mass or electrical resistance.

P. 768

**Other unit systems**

In fundamental theoretical physics another system is also common. One aim of research being the calculation of the strength of all interactions, setting the gravitational constant  $G$  to unity, as is done when using Planck units, makes this aim more difficult to express in equations. Therefore one often only sets  $c = \hbar = k = 1$  and  $\mu_0 = 1/\epsilon_0 = 4\pi$ ,\* leaving

\* Other definitions for the proportionality constants in electrodynamics lead to the Gaussian unit system often used in theoretical calculations, the Heaviside–Lorentz unit system, the electrostatic unit system, and the electromagnetic unit system, among others. For more details, see the standard text by JOHN DAVID JACKSON, *Classical Electrodynamics*, 3rd edition, Wiley, 1998.

only the gravitational constant  $G$  in the equations. In this system, only one fundamental unit exists, but its choice is still free.

Often a standard length is chosen as fundamental unit, length being the archetype of a measured quantity. The most important physical observables are related by

$$\begin{aligned} [l] &= 1/[E] = [t] = [C] = [L] \quad , \\ 1/[l] &= [E] = [m] = [p] = [a] = [f] = [I] = [U] = [T] \quad , \\ [l]^2 &= 1/[E]^2 = [G] = [P] = 1/[B] = 1/[E_{\text{el}}] \quad \text{and} \\ 1 &= [v] = [q] = [e] = [R] = [S_{\text{action}}] = [S_{\text{entropy}}] = \hbar = c = k = [\alpha] \end{aligned} \quad (665)$$

with the usual convention to write  $[x]$  for the unit of quantity  $x$ . Using the same unit for speed and electric resistance is not to everybody's taste, however, and therefore electricians do not use this system.\*

In many situations, in order to get an impression of the energies needed to observe the effect under study, a standard energy is chosen as fundamental unit. In particle physics the common energy unit is the *electron Volt* (eV), defined as the kinetic energy acquired by an electron when accelerated by an electrical potential difference of 1 Volt ('proton Volt' would be a better name). Therefore one has  $1 \text{ eV} = 1.6 \cdot 10^{-19} \text{ J}$ , or roughly

$$1 \text{ eV} \approx \frac{1}{6} \text{ aJ} \quad (666)$$

which is easily remembered. The simplification  $c = \hbar = 1$  yields  $G = 6.9 \cdot 10^{-57} \text{ eV}^{-2}$  and allows to use the unit eV also for mass, momentum, temperature, frequency, time and length, with the respective correspondences  $1 \text{ eV} \hat{=} 1.8 \cdot 10^{-36} \text{ kg} \hat{=} 5.4 \cdot 10^{-28} \text{ Ns} \hat{=} 242 \text{ THz} \hat{=} 11.6 \text{ kK}$  and  $1 \text{ eV}^{-1} \hat{=} 4.1 \text{ fs} \hat{=} 1.2 \mu\text{m}$ .

Challenge 1301 e

To get some feeling for the unit eV, the following relations are useful. Room temperature, usually taken as  $20^\circ \text{C}$  or  $293 \text{ K}$ , corresponds to a kinetic energy per particle of  $0.025 \text{ eV}$  or  $4.0 \text{ zJ}$ . The highest particle energy measured so far is a cosmic ray of energy of  $3 \cdot 10^{20} \text{ eV}$  or  $48 \text{ J}$ . Down here on the earth, an accelerator with an energy of about  $105 \text{ GeV}$  or  $17 \text{ nJ}$  for electrons and antielectrons has been built, and one with an energy of  $10 \text{ TeV}$  or  $1.6 \mu\text{J}$  for protons will be built soon. Both are owned by CERN in Geneva and have a circumference of  $27 \text{ km}$ .

Ref. 978

The lowest temperature measured up to now is  $280 \text{ pK}$ , in a system of Rhodium nuclei inside a special cooling system. The interior of that cryostat possibly is the coolest point in the whole universe. At the same time, the kinetic energy per particle corresponding to that temperature is also the smallest ever measured; it corresponds to  $24 \text{ feV}$  or  $3.8 \text{ vJ} = 3.8 \cdot 10^{-33} \text{ J}$ . For isolated particles, the record seems to be for neutrons: kinetic energies as low as  $10^{-7} \text{ eV}$  have been achieved, corresponding to De Broglie wavelengths of  $60 \text{ nm}$ .

Ref. 979

\* The web page <http://www.chemie.fu-berlin.de/chemistry/general/units-en.html> allows to convert various units into each other.

In general relativity still another system is sometimes used, in which the *Schwarzschild radius* defined as  $r_S = 2Gm/c^2$  is used to measure masses, by setting  $c = G = 1$ . In this case, in opposition to above, mass and length have the same dimension, and  $\hbar$  has dimension of an area.

### Curiosities

Here are a few facts making the concept of unit more vivid.

- A gray is the amount of radioactivity that deposits 1 J on 1 kg of matter. A sievert is a unit adjusted to human scale, where the different types of human tissues are weighted with a factor describing the effectiveness of radiation deposition. Four to five sievert are a lethal dose to humans. In comparison, the natural radioactivity present inside human bodies leads to a dose of 0.2 mSv per year. An average X-ray image is an irradiation of 1 mSv; a CAT scan 8 mSv.

Ref. 980

- Are you confused by the candela? The definition simply says that  $683 \text{ cd} = 683 \text{ lm/sr}$  correspond to  $1 \text{ W/sr}$ . The candela is thus a unit for light power per angle, except that it is corrected for the eye's sensitivity: the candela measures only *visible* power per angle. Similarly,  $683 \text{ lm} = 683 \text{ cd} \cdot \text{sr}$  correspond to  $1 \text{ W}$ , i.e. both the lumen and the watt measure power, or energy flux, except that the lumen measures only the *visible* part of the power. In English quantity names, the change is expressed by substituting 'radiant' by 'luminous'; e.g. the Watt measures *radiant* flux, whereas the lumen measure *luminous* flux.

The factor 683 is historical. A usual candle indeed emits a luminous intensity of about a candela. Therefore, at night, a candle can be seen up to a distance of one or two dozen kilometres. A 100 W incandescent light bulb produces 1700 lm and the brightest light emitting diodes about 5 lm.

Challenge 1302 e

The *irradiance* of sunlight is about  $1300 \text{ W/m}^2$  on a sunny day; the *illuminance* is  $120 \text{ klm/m}^2 = 120 \text{ klux}$  or  $170 \text{ W/m}^2$ . The numbers show that most energy radiated from the sun to the earth is outside the visible spectrum.

On a glacier, near the sea shore, on the top of mountains, or under particular weather condition the brightness can reach 150 klux. Lamps used during surgical operations usually produce 120 klux; humans need at least 100 lux for reading, but water paintings are destroyed by more than 100 lux, oil paintings by more than 200 lux. The full moon produces 0.1 lux; the eyes lose their ability to distinguish colours somewhere between 0.1 lux and 0.01 lux. The human body itself shines with about 1 plux, a value too small to be detected with the eye, but easily measured with apparatuses. The origin is unclear.

- The highest achieved light intensities are in excess of  $10^{18} \text{ W/m}^2$ , more than 15 orders of magnitude higher than the intensity of sunlight, and are achieved by tight focusing of pulsed lasers. The electric fields in such light pulses is of the same order of the field inside atoms; such a beam ionizes all matter it encounters.

Ref. 981

- The Planck length is roughly the de Broglie wavelength  $\lambda_B = h/mv$  of a man walking comfortably ( $m = 80 \text{ kg}$ ,  $v = 0.5 \text{ m/s}$ ); this motion is therefore aptly called the 'Planck stroll.'

Ref. 982

- The Planck mass is equal to the mass of about  $10^{19}$  protons. This is roughly the mass of a human embryo at about ten days of age.

- The second does not correspond to 1/86 400th of the day any more (it did so in the year 1900); the earth now takes about 86 400.002 s for a rotation, so that regularly the *International Earth Rotation Service* introduces a leap second to ensure that the sun is at

the highest point in the sky at 12.00 o'clock sharp. \* The time so defined is called *Universal Time Coordinate*. The velocity of rotation of the earth also changes irregularly from day to day due to the weather; the average rotation speed even changes from winter to summer due to the change in polar ice caps and in addition that average decreases over time, due to the friction produced by the tides. The rate of insertion of leap seconds is therefore faster than every 500 days, and not completely constant in time.

- The most precisely measured quantities in nature are the frequency of certain millisecond pulsars, \*\* the frequency of certain narrow atomic transitions and the Rydberg constant of *atomic* hydrogen, which can all be measured as exactly as the second is defined. At present, this gives about 14 digits of precision.

- The most precise clock ever built, using microwaves, had a stability of  $10^{-16}$  during a running time of 500 s. For longer time periods, the record in 1997 was about  $10^{-15}$ ; but the area of  $10^{-17}$  seems within technological reach. The precision of clocks is limited for short measuring times by noise and for long measuring times by drifts, i.e. by systematic effects. The region of highest stability depends on the clock type and usually lies between 1 ms for optical clocks and 5000 s for masers. Pulsars are the only clock for which this region is not known yet; it lies at more than 20 years, which is the time elapsed since their discovery.

Ref. 983

Ref. 984

- The shortest times measured are the life times of certain 'elementary' particles; in particular, the D meson was measured to live less than  $10^{-22}$  s. Such times are measured in a bubble chamber, where the track is photographed. Can you estimate how long the track is? (Watch out – if your result cannot be observed with an optical microscope, you made a mistake in your calculation).

Ref. 985

Challenge 1303 n

- The longest measured times are the lifetimes of certain radioisotopes, over  $10^{15}$  years, and the lower limit of certain proton decays, over  $10^{32}$  years. These times are thus much larger than the age of the universe, estimated to be fourteen thousand million years.

Ref. 986

- The least precisely measured fundamental quantities are the gravitational constant  $G$  and the strong coupling constant  $\alpha_s$ . Other, even less precisely known quantities, are the age of the universe and its density (see the astrophysical table below).

P. 942

- The precision of mass measurements of solids is limited by such simple effects as the adsorption of water on the weight. Can you estimate what a monolayer of water does on a weight of 1 kg?

Challenge 1304 n

- Variations of quantities are often much easier to measure than their values. For example, in gravitational wave detectors, the sensitivity achieved in 1992 was  $\Delta l/l = 3 \cdot 10^{-19}$  for lengths of the order of 1 m. In other words, for a block of about a cubic metre of metal it is possible to measure length changes about 3000 times smaller than a proton radius. These set-ups are now being superseded by ring interferometers. Ring interferometers measuring frequency differences of  $10^{-21}$  have already been built; they are still being improved towards higher values.

Ref. 988

\* Their web site at <http://hpiers.obspm.fr> gives more information on the details of these insertions, as does <http://maia.usno.navy.mil>, one of the few useful military web sites. See also <http://www.bipm.fr>, the site of the BIPM.

\*\* An overview of this fascinating work is given by J.H. TAYLOR, *Pulsar timing and relativistic gravity*, Philosophical Transactions of the Royal Society, London A **341**, pp. 117–134, 1992.

▪ The Swedish astronomer Anders Celsius (1701–1744) originally set the freezing point at 100 degrees and the boiling point of water at 0 degrees. Then the numbers were switched to get today's scale, with a small detail though. With the official definition of the Kelvin and the degree Celsius, at the standard pressure of 1013.25 Pa, water boils at 99.974 °C. Can you explain why it is not 100 °C any more?

Ref. 989

Challenge 1305 n

▪ The size of SI units is adapted to humans: heartbeat, human size, human weight, human temperature, human substance, etc. In a somewhat unexpected way they realise the saying by Protagoras, 25 centuries ago: 'Man is the measure of all things.'

▪ The table of SI prefixes covers seventy-two measurement decades. How many additional prefixes will be needed? Even an extended list will include only a small part of the infinite range of decades. Will the Conférence Générale des Poids et Mesures have to go on and on, defining an infinite number of SI prefixes?

Challenge 1306 n

▪ It is well-known that the French philosopher Voltaire, after meeting Newton, publicised the now famous story that the connection between the fall of objects and the motion of the moon was discovered by Newton when he saw an apple falling from a tree. More than a century later, just before the French revolution, a committee of scientists decided to take as unit of force precisely the force exerted by gravity on a *standard apple*, and to name it after the English scientist. After extensive study, it was found that the mass of the standard apple was 101.9716 g; its weight was called 1 newton. Since then, in the museum in Sèvres near Paris, visitors can admire the standard metre, the standard kilogram and the standard apple.\*

### Precision and accuracy of measurements

As explained on page 199, *precision* measures how well a result is reproduced when the measurement is repeated; *accuracy* is the degree to which a measurement corresponds to the actual value. Lack of precision is due to accidental or *random errors*; they are best measured by the *standard deviation*, usually abbreviated  $\sigma$ ; it is defined through

$$\sigma^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2 \quad (667)$$

where  $\bar{x}$  is the average of the measurements  $x_i$ . (Can you imagine why  $n-1$  is used in the formula instead of  $n$ ?) By the way, for a Gaussian distribution,  $2.35 \sigma$  is the full width at half maximum.

Challenge 1307 n

Lack of accuracy is due to *systematic errors*; usually they can only be estimated. This estimate is often added to the random errors to produce a *total* experimental error, sometimes also called *total uncertainty*.

Ref. 991

The following tables give the values of the most important physical constants and particle properties in SI units and in a few other common units, as published in the standard references. The values are the world average of the best measurements up to December

Ref. 992

\* To be clear, this is a joke; no standard apple exists. In contrast to the apple story it is *not* a joke however, that owners of several apple trees in Britain and in the US claim descent, by rerooting, from the original tree under which Newton had his insight. DNA tests have even been performed to decide if all these derive from the same tree, with the result that the tree at MIT, in contrast to the British ones, is a fake – of course.

Ref. 990

1998. As usual, experimental errors, including both random and estimated systematic errors, are expressed by giving the one standard deviation uncertainty in the last digits; e.g. 0.31(6) means – roughly speaking –  $0.31 \pm 0.06$ . In fact, behind each of the numbers in the following tables there is a long story which would be worth telling, but for which there is not enough room here.\*

What are the limits to accuracy and precision? First of all, there is no way, even in principle, to measure a quantity  $x$  to a *precision* higher than about 61 digits, because  $\Delta x/x \gtrsim l_{\text{Pl}}/d_{\text{horizon}} = 10^{-61}$ . In the third part of our text, studies of clocks and meter bars will further reduce this theoretical limit.

P. 802

But it is not difficult to deduce more stringent practical limits. No reasonable machine can measure quantities with a higher precision than measuring the diameter of the earth within the smallest length ever measured, about  $10^{-19}$  m; that makes about 26 digits. Using a more realistic limit of a 1000 m sized machine implies a limit of 22 digits. If, as predicted above, time measurements really achieve 17 digits of precision, then they are nearing the practical limit, because apart from size, there is an additional practical restriction: cost. Indeed, an additional digit in measurement precision means often an additional digit in equipment cost.

**Basic physical constants**

In principle, all experimental measurements of matter properties, such as colour, density, or elastic properties, can be predicted using the values of the following constants, using them in quantum theory calculations. Specifically, this is possible using the equations of the standard model of high energy physics.

Ref. 992

P. ??

**Table 70** Basic physical constants

Quantity	name	value in SI units	uncertainty
vacuum speed of light <sup>a</sup>	$c$	299 792 458 m/s	0
vacuum number of space-time dimensions		3 + 1 down to $10^{-19}$ m, up to $10^{26}$ m	
vacuum permeability <sup>a</sup>	$\mu_0$	$4\pi \cdot 10^{-7}$ H/m	0
		= 1.256 637 061 435 917 295 385 ... $\mu\text{H/m}$	
vacuum permittivity <sup>a</sup>	$\epsilon_0 = 1/\mu_0 c^2$	8.854 187 817 620 ... pF/m	0
Planck constant	$h$	$6.626\,068\,76(52) \cdot 10^{-34}$ Js	$7.8 \cdot 10^{-8}$
reduced Planck constant	$\hbar$	$1.054\,571\,596(82) \cdot 10^{-34}$ Js	$7.8 \cdot 10^{-8}$
positron charge	$e$	0.160 217 646 2(63) aC	$3.9 \cdot 10^{-8}$
Boltzmann constant	$k$	$1.380\,650\,3(24) \cdot 10^{-23}$ J/K	$1.7 \cdot 10^{-6}$
gravitational constant	$G$	$6.673(10) \cdot 10^{-11}$ Nm <sup>2</sup> /kg <sup>2</sup>	$1.5 \cdot 10^{-3}$
gravitational coupling constant	$\kappa = 8\pi G/c^4$	$2.076(3) \cdot 10^{-43}$ s <sup>2</sup> /kg m	$1.5 \cdot 10^{-3}$
fine structure constant, <sup>b</sup>	$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c}$	1/137.035 999 76(50)	$3.7 \cdot 10^{-9}$
e.m. coupling constant	$= g_{\text{em}}(m_e^2 c^2)$	= 0.007 297 352 533(27)	$3.7 \cdot 10^{-9}$

\* Some of them can be found in the text by N. W. WISE, *The Values of Precision*, Princeton University Press, 1994. The field of high precision measurements, from which the results on these pages stem, is a very special world. A beautiful introduction to it is *Near Zero: Frontiers of Physics*, edited by J.D. FAIRBANKS, B.S. DEEVER, C.W. EVERITT & P.F. MICHAELSON, Freeman, 1988.

Quantity	name	value in SI units	uncertainty
Fermi coupling constant, <sup>b</sup>	$G_F/(\hbar c)^3$	$1.166\,39(1) \cdot 10^{-5} \text{ GeV}^{-2}$	$8.6 \cdot 10^{-6}$
weak coupling constant	$\alpha_w(M_Z) = g_w^2/4\pi$	$1/30.1(3)$	
weak mixing angle	$\sin^2 \theta_W(\overline{MS})$	$0.231\,24(24)$	$1.0 \cdot 10^{-3}$
weak mixing angle	$\sin^2 \theta_W$ (on shell) $= 1 - (m_W/m_Z)^2$	$0.2224(19)$	$8.7 \cdot 10^{-3}$
strong coupling constant <sup>b</sup>	$\alpha_s(M_Z) = g_s^2/4\pi$	$0.118(3)$	$25 \cdot 10^{-3}$

a. Defining constant.

b. All coupling constants depend on the four-momentum transfer, as explained in the section on renormalisation. *Fine structure constant* is the traditional name for the electromagnetic coupling constant  $g_{em}$  in the case of a four momentum transfer of  $Q^2 = m_e^2 c^2$ , which is the smallest one possible. At higher momentum transfers it has larger values, e.g.  $g_{em}(Q^2 = M_W^2 c^2) \approx 1/128$ . The strong coupling constant has higher values at lower momentum transfers; e.g. one has  $\alpha_s(34 \text{ GeV}) = 0.14(2)$ .

Why do all these constants have the values they have? The answer depends on the constant. For any constant having a unit, such as the quantum of action  $\hbar$ , the numerical value has no intrinsic meaning. It is  $1.054 \cdot 10^{-34} \text{ Js}$  because of the SI definition of the joule and the second.

However, the question why the value of a constant with units is not larger or smaller always requires to understand the origin of some dimensionless number. For example,  $\hbar$ ,  $G$  and  $c$  are not smaller or larger because the everyday world, in basic units, is of the dimensions we observe. The same happens if we ask about the size of atoms, people, trees and stars, about the duration of molecular and atomic processes, or about the mass of nuclei and mountains. Understanding the values of all dimensionless constants is thus the key to understanding nature.

The basic constants yield the following useful high-precision observations.

**Table 71** Derived physical constants

Quantity	name	value in SI units	uncertainty
Vacuum wave resistance	$Z_0 = \sqrt{\mu_0/\epsilon_0}$	$376.730\,313\,461\,77\dots \Omega$	0
Avogadro's number	$N_A$	$6.022\,141\,99(47) \cdot 10^{23}$	$7.9 \cdot 10^{-8}$
Rydberg constant <sup>a</sup>	$R_\infty = m_e c \alpha^2 / 2h$	$10\,973\,731.568\,549(83) \text{ m}^{-1}$	$7.6 \cdot 10^{-12}$
mag. flux quantum	$\varphi_0 = h/2e$	$2.067\,833\,636(81) \text{ pWb}$	$3.9 \cdot 10^{-8}$
Josephson freq. ratio	$2e/h$	$483.597\,898(19) \text{ THz/V}$	$3.9 \cdot 10^{-8}$
von Klitzing constant	$h/e^2 = \mu_0 c / 2\alpha$	$25\,812.807\,572(95) \Omega$	$3.7 \cdot 10^{-9}$
Bohr magneton	$\mu_B = e\hbar/2m_e$	$9.274\,008\,99(37) \cdot 10^{-24} \text{ J/T}$	$4.0 \cdot 10^{-8}$
classical electron radius	$r_e = e^2/4\pi\epsilon_0 m_e c^2$	$2.817\,940\,285(31) \text{ fm}$	$1.1 \cdot 10^{-8}$
Compton wavelength of the electron	$\lambda_c = h/m_e c$	$2.426\,310\,215(18) \text{ pm}$	$7.3 \cdot 10^{-9}$
	$\lambda_c = \hbar/m_e c = r_e/\alpha$	$0.386\,159\,264\,2(28) \text{ pm}$	$7.3 \cdot 10^{-9}$
Bohr radius <sup>a</sup>	$a_\infty = r_e/\alpha^2$	$52.917\,720\,83(19) \text{ pm}$	$3.7 \cdot 10^{-9}$
cyclotron frequency	$f_c/B = e/2\pi m_e$	$27.992\,4925(11) \text{ GHz/T}$	$4.0 \cdot 10^{-8}$

Quantity	name	value in SI units	uncertainty
of the electron			
nuclear magneton	$\mu_N = e\hbar/2m_p$	$5.050\,783\,17(20) \cdot 10^{-27}$ J/T	$4.0 \cdot 10^{-8}$
proton electron mass ratio	$m_p/m_e$	1 836.152 667 5(39)	$2.1 \cdot 10^{-9}$
Stefan–Boltzmann constant	$\sigma = \pi^2 k^4 / 60\hbar^3 c^2$	$5.670\,400(40) \cdot 10^{-8}$ W/m <sup>2</sup> K <sup>4</sup>	$7.0 \cdot 10^{-6}$
Wien displacement law constant	$b = \lambda_{\max} T$	2.897 7686(51) mmK	$1.7 \cdot 10^{-6}$
bits to entropy conv. const.		$10^{23}$ bit = 0.956 994 5(17) J/K	
TNT energy content		3.7 to 4.0 MJ/kg = $4 \cdot 10^3$ m <sup>2</sup> /s <sup>2</sup>	

a. For infinite mass of the nucleus.

Some properties of the universe as a whole are listed in the following.

**Table 72** Astrophysical constants

Quantity	name	value
gravitational constant	$G$	$6.672\,59(85) \cdot 10^{-11}$ m <sup>3</sup> /kg s <sup>2</sup>
cosmological constant	$\Lambda$	ca. $1 \cdot 10^{-52}$ m <sup>-2</sup>
tropical year 1900 <sup>a</sup>	$a$	31 556 925.974 7 s
tropical year 1994	$a$	31 556 925.2 s
mean sidereal day	$d$	$23^h 56^m 4.090\,53''$
astronomical unit <sup>b</sup>	AU	149 597 870.691(30) km
light year	al	9.460 528 173 ... Pm
parsec	pc	30.856 775 806 Pm = 3.261 634 al
age of the universe <sup>c</sup>	$t_0$	$> 3.5(4) \cdot 10^{17}$ s or $> 11.5(1.5) \cdot 10^9$ a (from matter, via galaxies and stars, using quantum theory: early 1997 results)
age of the universe <sup>c</sup>	$t_0$	$4.32(7) \cdot 10^{17}$ s = $13.7(2) \cdot 10^9$ a (from space-time, via expansion, using general relativity)
universe's horizon's dist. <sup>c</sup>	$d_0 = 3ct_0$	$5.2(1.4) \cdot 10^{26}$ m = 13.8(4.5) Gpc
universe's topology		unknown
number of space dimensions		3
Hubble parameter <sup>c</sup>	$H_0$	$2.2(1.0) \cdot 10^{-18}$ s <sup>-1</sup> = $0.7(3) \cdot 10^{-10}$ a <sup>-1</sup> = $h_0 \cdot 100$ km/sMpc = $h_0 \cdot 1.0227 \cdot 10^{-10}$ a <sup>-1</sup>
reduced Hubble par. <sup>c</sup>	$h_0$	$0.59 < h_0 < 0.7$
critical density	$\rho_c = 3H_0^2/8\pi G$	$h_0^2 \cdot 1.878\,82(24) \cdot 10^{-26}$ kg/m <sup>3</sup>
of the universe		
density parameter <sup>c</sup>	$\Omega_{M_0} = \rho_0/\rho_c$	ca. 0.3
luminous matter density		ca. $2 \cdot 10^{-28}$ kg/m <sup>3</sup>
stars in the universe	$n_s$	$10^{22\pm 1}$
baryons in the universe	$n_b$	$10^{81\pm 1}$
baryon mass	$m_b$	$1.7 \cdot 10^{-27}$ kg
baryon number density		1 to 6 /m <sup>3</sup>
photons in the universe	$n_\gamma$	$10^{89}$
photon energy density	$\rho_\gamma = \pi^2 k^4 / 15T_0^4$	$4.6 \cdot 10^{-31}$ kg/m <sup>3</sup>
photon number density		$400/\text{cm}^3 (T_0/2.7\text{ K})^3$ , at present 410.89/cm <sup>3</sup>
background temperature <sup>d</sup>	$T_0$	2.726(5) K

Quantity	name	value
Planck length	$l_{\text{Pl}} = \sqrt{\hbar G/c^3}$	$1.62 \cdot 10^{-35}$ m
Planck time	$t_{\text{Pl}} = \sqrt{\hbar G/c^5}$	$5.39 \cdot 10^{-44}$ s
Planck mass	$m_{\text{Pl}} = \sqrt{\hbar c/G}$	21.8 $\mu\text{g}$
instants in history <sup>c</sup>	$t_o/t_{\text{Pl}}$	$8.7(2.8) \cdot 10^{60}$
space-time points	$N_o = (R_o/l_{\text{Pl}})^3$	$10^{244 \pm 1}$
inside the horizon <sup>c</sup>	$(t_o/t_{\text{Pl}})$	
mass inside horizon	$M$	$10^{54 \pm 1}$ kg

Challenge 1308 e  
Ref. 993

a. Defining constant, from vernal equinox to vernal equinox; it was once used to define the second. (Remember:  $\pi$  seconds is a nanocentury.) The value for 1990 is about 0.7 s less, corresponding to a slowdown of roughly  $-0.2$  ms/a. (Why?) There is even an empirical formula available for the change of the length of the year over time.

b. Average distance earth–sun. The truly amazing precision of 30 m results from time averages of signals sent from Viking orbiters and Mars landers taken over a period of over twenty years.

c. The index o indicates present day values.

d. The radiation originated when the universe was between  $10^5$  to  $10^6$  years old and about 3000 K hot; the fluctuations  $\Delta T_o$  which lead to galaxy formation are today of the size of  $16 \pm 4 \mu\text{K} = 6(2) \cdot 10^{-6} T_o$ .

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Attention: in the third part of this text it is shown that many constants in Table 72 are *not* physically sensible quantities. They have to be taken with lots of grains of salt. The more specific constants given in the following table are all sensible though.

**Table 73** Astronomical constants

Quantity	name	value
earth’s mass	$M$	$5.972\,23(8) \cdot 10^{24}$ kg
earth’s gravitational length	$l = 2GM/c^2$	8.870(1) mm
earth radius, equatorial <sup>a</sup>	$R_{\text{eq}}$	6378.1367(1) km
earth’s radius, polar <sup>a</sup>	$R_{\text{p}}$	6356.7517(1) km
equator–pole distance <sup>a</sup>		10 001.966 km (average)
earth’s flattening <sup>a</sup>	$e$	1/298.25231(1)
earth’s av. density	$\rho$	5.5 Mg/m <sup>3</sup>
moon’s radius	$R_{\text{mv}}$	1738 km in direction of earth
moon’s radius	$R_{\text{mh}}$	17.. km in other two directions
moon’s mass	$M_{\text{m}}$	$7.35 \cdot 10^{22}$ kg
moon’s mean distance <sup>b</sup>	$d_{\text{m}}$	384 401 km
moon’s perigeon		typically 363 Mm, hist. minimum 359 861 km
moon’s apogeon		typically 404 Mm, hist. maximum 406 720 km
moon’s angular size <sup>c</sup>		avg. $0.5181^\circ = 31.08'$ , min. $0.49^\circ$ , max. $0.55^\circ$
moon’s av. density	$\rho$	3.3 Mg/m <sup>3</sup>
sun’s mass	$M_{\odot}$	$1.988\,43(3) \cdot 10^{30}$ kg
sun’s grav. length	$l_{\odot} = 2GM_{\odot}/c^2$	2.953 250 08 km
sun’s luminosity	$L_{\odot}$	384.6 YW
solar radius, equatorial	$R_{\odot}$	695.98(7) Mm

Quantity	name	value
sun's angular size		0.53° average; minimum on 4th of July (aphelion) 1888", maximum on 4th of January (perihelion) 1952"
sun's av. density	$\rho$	1.4 Mg/m <sup>3</sup>
sun's distance, average	AU	149 597 870.691(30) km
solar velocity around centre of galaxy	$v_{\odot g}$	220(20) km/s
solar velocity against cosmic background	$v_{\odot b}$	370.6(5) km/s
distance to galaxy centre		8.0(5) kpc = 26.1(1.6) kal
most distant galaxy	0140+326RD1	$12.2 \cdot 10^9$ al = $1.2 \cdot 10^{26}$ m, red-shift 5.34

a. The shape of the earth is described most precisely with the World Geodetic System. The last edition dates from 1984. For an extensive presentation of its background and its details, see the <http://www.eurocontrol.be/projects/eatchip/wgs84/start.html> web site. The International Geodesic Union has refined the data in 2000. The radii and the flattening given here are those for the 'mean tide system'. They differ from those of the 'zero tide system' and other systems by about 0.7 m. The details are a science by its own.

b. Measured centre to centre. To know the precise position of the moon at a given date, see the <http://www.fourmilab.ch/earthview/moon-ap-per.html> site, whereas for the planets see <http://www.fourmilab.ch/solar/solar.html> as well as the other pages on this site.

c. Angles are defined as follows: 1 degree =  $1^\circ = \pi/180$  rad, 1 (first) minute =  $1' = 1^\circ/60$ , 1 second (minute) =  $1'' = 1'/60$ . The ancient units 'third minute' and 'fourth minute', each 1/60th of the preceding, are not accepted any more. ('Minute' originally means 'very small', as it still does in modern English.)

### Useful numbers

$\pi$	3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510 <sub>5</sub>
$e$	2.71828 18284 59045 23536 02874 71352 66249 77572 47093 69995 <sub>9</sub>
$\gamma$	0.57721 56649 01532 86060 65120 90082 40243 10421 59335 93992 <sub>3</sub>
$\ln 2$	0.69314 71805 59945 30941 72321 21458 17656 80755 00134 36025 <sub>5</sub>
$\ln 10$	2.30258 50929 94045 68401 79914 54684 36420 76011 01488 62877 <sub>2</sub>
$\sqrt{10}$	3.16227 76601 68379 33199 88935 44432 71853 37195 55139 32521 <sub>6</sub>

Ref. 994

If the number  $\pi$  were *normal*, i.e. if all digits and digit combinations would appear with the same probability, then every text written or to be written, as well as every word spoken or to be spoken, can be found coded in its sequence. The property of normality has not yet been proven, even though it is suspected to be true. What is the significance? Is all wisdom encoded in the simple circle? No. The property is nothing special, as it also applies to the number 0.123456789101112131415161718192021... and many others. Can you specify a few?

Challenge 1309 n

By the way, in the graph of the exponential function  $e^x$ , the point (0, 1) is the only one with two rational coordinates. If you imagine to paint in blue all points on the plane with two rational coordinates, the plane would look quite bluish. Nevertheless, the graph goes only through one of these points and manages to avoid all the others.



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- 993** The details are given in the well-known astronomical reference, P. KENNETH SEIDELMANN, *Explanatory Supplement to the Astronomical Almanac*, 1992. Cited on page 943.
- 994** For information about the number  $\pi$ , as well as about other constants, the web address <http://www.cecm.sfu.ca/pi/pi.html> provides lots of data and references. It also has a link to the pretty overview paper on <http://www.astro.virginia.edu/~eww6n/math/Pi.html> and to many other sites on the topic. Simple formulae for  $\pi$  are

$$\pi + 3 = \sum_{n=1}^{\infty} \frac{n 2^n}{\binom{2n}{n}} \quad (668)$$

or the beautiful formula discovered in 1996 by Bailey, Borwein and Plouffe

$$\pi = \sum_{n=0}^{\infty} \frac{1}{16^n} \left( \frac{4}{8n+1} - \frac{2}{8n+4} - \frac{1}{8n+5} - \frac{1}{8n+6} \right) . \quad (669)$$

The site also explains the newly discovered methods to calculate specific binary digits of  $\pi$  without having to calculate all the preceding ones. By the way, the number of (consecutive) digits known in 1999 was over 1.2 million million, as told in Science News **162**, 14 December 2002. They pass all tests for a random string of numbers, as the [http://www.ast.univie.ac.at/~wasi/PI/pi\\_normal.html](http://www.ast.univie.ac.at/~wasi/PI/pi_normal.html) web site explains. However, this property, called *normality*, has never been proven; it is the biggest open question about  $\pi$ . It is possible that the theory of chaotic dynamics will lead to a solution of this puzzle in the coming years.

Another method to calculate  $\pi$  and other constants was discovered and published by DAVID V. CHUDNOVSKY & GREGORY V. CHUDNOVSKY, *The computation of classical constants*, Proc. Natl. Acad. Sci. USA, volume 86, pp. 8178–8182, 1989. The Chudnowsky brothers have built a supercomputer in Gregory's apartment for about 70 000 Euro, and for many years held the record for the largest number of digits for  $\pi$ . They battle already for decades with Kanada Yasumasa, who holds the record in 2000, calculated on an industrial supercomputer. New formulae to calculate  $\pi$  are still irregularly discovered.

For the calculation of Euler's constant  $\gamma$  see also D.W. DETEMPLE, *A quicker convergence to Euler's constant*, The Mathematical Intelligencer, pp. 468–470, May 1993.

Note that little is known about properties of numbers; e.g. it is still not known whether  $\pi + e$  is a rational number or not! (It is believed that it is not.) Do you want to become a mathematician? Cited on page 944.

Challenge 1310 r  
Challenge 1311 n

